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Technical Note TN 27

Three ALGOL - 60 programs

Complex Arithmetic,
Difference and Derivative procedures,
Determination of the point of Toricelli

by

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In this note some peculiar ALGOL - 60 procedures are described. The first gives a method to build up arbitrary arithmetic expressions with complex variables.

The other describes a method for calculating forward differences and derivatives from a given set of function-values with equally spaced parameter-values. It makes use of strong recursive procedures.

We do not claim that this method is handsome for numerical use, on the contrary, as we observed with the X1 computer of the Mathematical Centre, it works of course, but wastes awfully much time. We find it however a very beautiful and instructive illustration of the capability of ALGOL - 60.

We now describe the first procedures.

```

procedure Complex Arithmetic (a, b, R, r); array a, b, R, r;
begin   integer i, k; array H [1:4, 1:2];
        integer procedure P (i, j); value i, j; integer i, j;
        begin real a; k:= k - 1; a:= H [i, 1] × H [j, 1] - H [i, 2] ×
            H [j, 2]; H [k, 2]:= H [i, 1] × H [j, 2] + H [i, 2] ×
            H [j, 1]; H [k, 1]:= a; P:= k
        end;
        integer procedure Q (i, j); value i, j; integer i, j;
        begin real a, b; k:= k - 1; b:= H [j, 1]√2 + H [j, 2]√2;
            a:= (H [i, 1] × H [j, 1] + H [i, 2] × H [j, 2])/b;
            H [k, 2]:= (H [i, 2] × H [j, 1] - H [i, 1] × H [j, 2])/b;
            H [k, 1]:= a; Q:= k
        end;
        integer procedure S (i, j); value i, j; integer i, j;
        begin k:= k - 1; H [k, 1]:= H [i, 1] + H [j, 1]; H [k, 2]:=
            H [i, 2] + H [j, 2]; S:= k
        end;
        integer procedure T (a); array a;
        begin k:= k + 1; H [k, 1]:= a [1]; H [k, 2]:= a [2]; T:= k end;
        integer procedure J (i, expi); integer i; real expi;
        begin k:= k + 1; i:= 1; H [k, 1]:= expi; i:= 2; H [k, 2]:= expi;
            J:= k
        end;
        procedure U (i, R); value i; integer i; array R;
        begin R [1]:= H [i, 1]; R [2]:= H [i, 2]; k:= 0 end;

        k:= 0;
        U (S (P (T (a))times:(T (a)))plus:(P (T (b))times:(T (b))), R);
        comment (    a    ×        a    ) +        (    b    ×        b    )=: R ;

        U (Q (S (T (a))plus:(P (J (i,1-1))times:(T (b))))divided by:
            (S (T (a))plus:(P (J (i,1-1))times:(T (b)))), r);
        comment (    a    +        (    i        ×        b    )    /
            (    a    +        (    -i        ×        b    ))=: r
    
```

end

This procedure assigns the value $a\sqrt{2} + b\sqrt{2}$ to R and the value $(a + ib)/(a - ib)$ to r, where a, b, R and r are complex numbers.

We focus attention to the value call of the parameters of the procedures, this is essential.

Furthermore we notice that the depth or height of the accumulator H is the maximum number of right-handed brackets placed one after another, not calculating the brackets which occur in parameter-delimiters.

The following program calculates the third derivative of $\exp(x)$ with a 6-point difference scheme, the error is of the order h^3 , where h is the steplength.

```

begin  real h; integer i, k; array A [1:50];
      real procedure SUM(i,h,k,ti); value k; integer i,k,h; real ti;
      begin real s; s:= 0; for i:= h step 1 until k do s:= s + ti;
        SUM:= s
      end;
      real procedure DELTA (N,k,k0,fk); value N,k0; real fk;
      integer N,k,k0;
      comment N is the order of the forward difference which is
      calculated from a set of function-values with equidistant
      parameter-values, in formula:

              DELTA =  $\Delta^N f(k_0)$  ;
begin  integer i;
      DELTA:= if N = 1
      then SUM (k,k0,k0+1,(-1)^(k+1-k0) × fk)
      else DELTA (1,i,k0,DELTA (N-1,k,i,fk))
end;
      real procedure DER (OR,N,h,k,k0,fk); value OR,N,h,k0; real fk,h;
      integer OR,N,k,k0;
      comment OR is the order of the derivative, calculated from a given
      set of function-values  $f(k)$ , with equidistant parameter-values,
      the error is of the order  $h^{(N+1-OR)}$ , where  $h$  is the steplength,
       $k_0$  is the point where the derivative is calculated, in formula:

      DER:=  $(-h)^{(OR)} \sum_{i_1, \dots, i_{OR}=1}^{N+1-OR} (-\Delta)^{(i_1+\dots+i_{OR})} / (i_1 \times \dots \times i_{OR}) \times f(k_0)$ ;

begin  integer i;
      DER:= if OR = 1
      then SUM (i,1,N,DELTA (i,k,k0,fk) × (-1)^(i+1)/i)/h
      else DER (1,N+1-OR,h,i,k0,DER (OR-1,N-1,h,k,i,fk))
end;

      for i:= 1 step 1 until 50 do A[i]:= exp (i/50);
      for i:= 1 step 1 until 25 do A[i]:= DER (3,6,1/50,k,i,A[k])
end

```

At the end of this program the third derivatives of $\exp(x)$ in the points $i = 1/50, 2/50, \dots, 25/50$ are assigned to the array $A[i]$.

Next follows a program which calculates from a given set of points $(x[i], y[i])$, $i = 1, 2, \dots, n$, the point (x_0, y_0) which has a minimal distance to all these points, called the point of Toricelli. Starting with some point (x_0, y_0) the program seeks automatically the best direction φ , where a better point will be found. Then (x_0, y_0) is replaced by $(x_0 + r \cos(\varphi), y_0 + r \sin(\varphi))$, where r is the steplength. Until the steplength r is smaller than some number ϵ the program will continue.

On a read-band the following numbers must be punched:
 The number of points,
 the first point (x_0, y_0) (in this sequence),
 the points $(x[i], y[i])$ (in this sequence),
 the first steplength,
 the number ϵ .

```

begin  real x0, y0, r, dx, dy, eps, S, C, RHO1, RHO2; integer i, k, n;
      n:= read; k:= 0;
      begin  array x1, x2, y1, y2, A, B [1:n];
            x0:= read; y0:= read; NLCR; PRINTTEXT (<Het startpunt is:>);
            print(x0); print (y0); NLCR;
            PRINTTEXT (<De hoekpunten zijn:>); NLCR; for i:= 1 step 1
            until n do
            begin x1 [i]:= read; print (x1 [i]); y1 [i]:= read; print
              (y1 [i]); x1 [i]:= x1 [i] - x0; y1 [i]:= y1 [i] - y0;
              A [i]:= sqrt (x1 [i]2 + y1 [i]2); NLCR
            end;
            r:= read; NLCR; PRINTTEXT (<De eerste staplengte is:>);
            print (r); NLCR; eps:= read; print (eps); NLCR; PRINTTEXT
            (<Het minimum punt is: De afstand is: Som sin is: Som cos is:>);
            NLCR; RHO1:= SUM (i, 1, n, A [i]);
            OPNIEUW: S:= SUM (i, 1, n, y1 [i]/A [i]); C:= SUM (i, 1, n, x1 [i]/
            A [i]); if S = 0 ^ C = 0 then goto EIND;
            dx:= sqrt (S2 + C2); dy:= S x r/dx; dx:= C x r/dx;
            goto LOOP DOOR;
            BEGIN: dx:= .1 x dx; dy:= .1 x dy; r:= .1 x r;
            LOOP DOOR: for i:= 1 step 1 until n do
            begin x2 [i]:= x1 [i] - dx; y2 [i]:= y1 [i] - dy; B [i]:=
            sqrt (x2 [i]2 + y2 [i]2) end;
            k:= k + 1; RHO2:= SUM (i, 1, n, B [i]); if r < eps then
            goto EIND; if RHO2 > RHO1 then goto BEGIN else
            begin for i:= 1 step 1 until n do
            begin x1 [i]:= x2 [i]; y1 [i]:= y2 [i];
              A [i]:= B [i]
            end; x0:= x0 + dx; y0:= y0 + dy; RHO1:= RHO2;
            print (k); print (x0); print (y0); print (RHO1);
            print (S); print (C); NLCR; goto OPNIEUW
            end;
      end;
EIND:  end
end

```